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A. I. International School

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Rational Number

What is a Rational Number?

A rational number, in Mathematics, can be defined as any number which can be represented in the form of p/q where q ≠ 0. Where p and q may be positive may be negative. Also, we can say that any fraction fits under the category of rational numbers, where the denominator and numerator are integers and the denominator is not equal to zero. When the rational number is divided, the result will be in decimal form, which may be either terminating decimal or the repeating decimal.

How to identify rational numbers?

To identify if a number is rational or not, check the below conditions.

- It is represented in the form of p/q, where q≠0.
- The ratio p/q can be further simplified and represented in decimal form.
- All whole numbers are rational numbers

Examples of Rational Numbers:-

p	q	p/q	Rational
10	2	10/2 =5	Rational
1	1000	1/1000 = 0.001	Rational
50	10	50/10 = 5	Rational

Types of Rational Numbers:-

A number is rational if we can write it as a fraction, where both denominator and numerator are integers and the denominator is a non-zero number.

- integers like -2, 0, 3 etc.
- fractions whose numerators and denominators are integers like 3/7, -6/5, etc.

- terminating decimals like 0.35, 0.7116, 0.9768, etc.
- non-terminating decimals with some repeating patterns (after the decimal point) such as 0.333..., 0.141414..., etc. These are popularly known as non-terminating repeating decimals.

<u>Standard Form of Rational Numbers:-</u>

The standard form of a rational number can be defined if it's no common factors aside from one between the dividend and divisor and therefore the divisor is positive.

For example, 12/36 is a rational number. But it can be simplified as 1/3; common factors between the divisor and dividend is only one. So we can say that rational number ⅓ is in standard form.

<u>Positive and Negative Rational Numbers:-</u>

As we know that the rational number is in the form of p/q, where p and q are integers. Also, q should be a non-zero integer. The rational number can be either positive or negative. If the rational number is positive, both p and q are positive integers. If the rational number takes the form -(p/q), then either p or q takes the negative value. It means that

-(p/q) = (-p)/q = p/(-q).

Now, let's discuss some of the examples of positive and negative rational numbers.

Positive Rational Numbers	**Negative Rational Numbers**
If both the numerator and denominator are of the same signs.	If numerator and denominator are of opposite signs.
All are greater than 0	All are less than 0
Examples of positive rational numbers: 12/17, 9/11 and 3/5	Examples of negative rational numbers: -2/17, 9/-11 and -1/5.

<u>Arithmetic Operations on Rational Numbers:-</u>

In Maths, arithmetic operations are the basic operations we perform on integers. Let us discuss here how we can perform these operations on rational numbers, say p/q and s/t.

<u>Addition:</u> When we add p/q and s/t, we need to make the denominator the same. Hence, we get (pt+qs)/qt.

Example: 1/2 + 3/4 = (2+3)/4 = 5/4

<u>Subtraction:</u> Similarly, if we subtract p/q and s/t, then also, we need to make the denominator same, first, and then do the subtraction.

Example: 1/2 – 3/4 = (2-3)/4 = -1/4

Multiplication: In case of multiplication, while multiplying two rational numbers, the numerator and denominators of the rational numbers are multiplied, respectively. If p/q is multiplied by s/t, then we get (p×s)/(q×t).

Example: 1/2 × 3/4 = (1×3)/(2×4) = 3/8

Division: If p/q is divided by s/t, then it is represented as:
(p/q)÷(s/t) = pt/qs

Example: 1/2 ÷ 3/4 = (1×4)/(2×3) = 4/6 = 2/3

Multiplicative Inverse of Rational Numbers:-

As the rational number is represented in the form p/q, which is a fraction, then the multiplicative inverse of the rational number is the reciprocal of the given fraction.

For example, 4/7 is a rational number, then the multiplicative inverse of the rational number 4/7 is 7/4, such that (4/7)x(7/4) = 1

Rational Numbers Properties:-

Since a rational number is a subset of the real number, the rational number will obey all the properties of the real number system. Some of the important properties of the rational numbers are as follows:

- The results are always a rational number if we multiply, add, or subtract any two rational numbers.
- A rational number remains the same if we divide or multiply both the numerator and denominator with the same factor.
- If we add zero to a rational number then we will get the same number itself.
- Rational numbers are closed under addition, subtraction, and multiplication.

Difference Between Irrational and Rational Numbers:-

The numbers which are NOT rational numbers are called irrational numbers. The set of irrational numbers is represented by Q´. The difference between rational and irrational numbers are as follows:

Rational Numbers	**Irrational Numbers**
These are numbers that can be expressed as fractions of integers. Examples: 0.75, -31/5, etc	These are numbers that CANNOT be expressed as fractions of integers. Examples: √5, π, etc.
They can be terminating decimals.	They are NEVER terminating decimals.
They can be non-terminating decimals with repetitive	They should be non-terminating decimals with NO repetitive patterns of decimals.

patterns of decimals. Example: 1.414, 414, 414 ... has repeating patterns of decimals where 414 is repeating.	Example: √5 = 2.236067977499789696409173.... has no repeating patterns of decimals
The set of rational numbers contains all-natural numbers, all whole numbers, and all integers.	The set of irrational numbers is a separate set and it does NOT contain any of the other sets of numbers.

<u>How to Find the Rational Numbers between Two Rational Numbers?</u>

There are "n" numbers of rational numbers between two rational numbers. The rational numbers between two rational numbers can be found easily using two different methods. Now, let us have a look at the two different methods.

Method 1:

Find out the equivalent fraction for the given rational numbers and find out the rational numbers in between them. Those numbers should be the required rational numbers.

Method 2:

Find out the mean value for the two given rational numbers. The mean value should be the required rational number. In order to find more rational numbers, repeat the same process with the old and the newly obtained rational numbers.

Example 1:

Identify each of the following as irrational or rational: ¾ , 90/12007, 12 and √5.

Solution:

Since a rational number is the one that can be expressed as a ratio. This indicates that it can be expressed as a fraction wherein both denominator and numerator are whole numbers.

- ¾ is a rational number as it can be expressed as a fraction. 3/4 = 0.75
- Fraction 90/12007 is rational.
- 12, also be written as 12/1. Again a rational number.
- Value of √5 = 2.2360679775…….. It is a non-terminating value and hence cannot be written as a fraction. It is an irrational number.

Example 2:

Identify whether mixed fraction, $1^1/_2$ is a rational number.

Solution:

The Simplest form of $1^1/_2$ is 3/2

Numerator = 3, which is an integer

Denominator = 2, is an integer and not equal to zero.

So, yes, 3/2 is a rational number.

Example 3:

Determine whether the given numbers are rational or irrational.

(a) 1.75 (b) 0.01 (c) 0.5 (d) 0.09 (d) √3

Solution:

The given numbers are in decimal format. To find whether the given number is decimal or not, we have to convert it into the fraction form (i.e., p/q)

If the denominator of the fraction is not equal to zero, then the number is rational, or else, it is irrational.

Decimal Number	Fraction	Rational Number
1.75	7/4	yes
0.01	1/100	yes
0.5	1/2	yes
0.09	1/11	yes
√ 3	?	No

<u>How to Calculate a Rational Number between two Rational Numbers?</u>

If 'a' and 'b' are the two rational numbers such that "a < b", then the rational number between 'a' and 'b' is calculated by using the formula: 1/2 (a + b)

<u>Solved Example:-</u>

Find the rational numbers between the two rational numbers 4/5 and 3/2.

Solution:

Here, a = 4/5 and b = 3/2

Now use the formula 1/2 (a + b) to calculate the rational numbers between 4/5 and 3/2.

Required Number = 1/2 (4/5 + 3/2)

= 1/2 ((8+15)/10)

= 1/2 ((23)/10)

= 23/20

Therefore, the rational number between 4/5 and 3/2 is 23/20.

Now, try to use the calculator and calculate a rational number between two rational numbers given below.

- 3/5 and 6/5
- 2/3 and 4/3

List of Rational Numbers:-

From the above information, it is clear that there is an infinite number of rational numbers. Hence, it is not possible to determine the list of rational numbers.

Smallest Rational Number:-

Since we cannot determine the list of rational numbers, we cannot determine the smallest rational number.

Rational Numbers Tips and Tricks:-

- Rational numbers are NOT only fractions but any number that can be expressed as fractions.
- Natural numbers, whole numbers, integers, fractions of integers, and terminating decimals are rational numbers.
- Non-terminating decimals with repeating patterns of decimals are also rational numbers.
- If a fraction has a negative sign either to the numerator or to the denominator or in front of the fraction, the fraction is negative. i.e, -a/b = a/-b.